The Venture Capital Premium: *A New Approach*

Luis E. Pereiro

Luis E. Pereiro
is a professor of corporate finance at HEC Paris School of Management and Universidad Torcuato Di Tella in Buenos Aires, Argentina.
lpereiro@utdt.edu

Despite venture capital (VC) being a $200+ billion-dollar industry, it is not yet clear what minimum return investors should demand on their VC investments.[1] Here is why: The portfolios of those who invest in public stocks are composed of minority stock positions in relatively seasoned, liquid companies; in contrast, the portfolios of venture capitalists (VCs) are usually composed of control positions in less well-established (read: more prone to fail) and private (read: less liquid) ventures. The VC asset class is thus riskier than public equity in three important ways:

Illiquidity: A large proportion of the companies that make up a VC portfolio do not percolate through the initial public offering (IPO) filter, and thus remain less marketable over a fund's lifetime than comparable public stock. Illiquidity decreases the value of a VC portfolio—in other words, it makes the portfolio riskier.

Hazard of failure. Along the VC investment cycle, a sizable segment of a portfolio inevitably goes under. Such probability of failure is material and has the effect, like illiquidity, of decreasing the value—namely, augmenting the risk—of a VC portfolio.

Control. In contrast to minority investors in public equity, VCs have the benefit of control: Their contracts with entrepreneurs contain clauses that ensure VCs' decision-making dominance, even if they hold minority positions in the target ventures. The ability to keep control over the decision-making process of portfolio companies increases the value of VCs' portfolios—namely, decreases its risk, somewhat compensating for the deleterious effects of hazard and illiquidity.

What premium should an investor add on top of public equity's return to get fair compensation for the peculiar risks of investing in venture capital? A few researchers have attempted to answer this question by using variations of the Capital Asset Pricing Model (CAPM). These attempts, alas, suffer from two serious flaws:

All CAPM-based models provide estimates of realized VC premiums, which, as such, may be reflecting compensation for risk and superior fund management. This makes it impossible to distinguish between the intrinsic risk of VC portfolios and the managerial ability of those who manage them.

Available CAPM-based models do not account for the illiquidity proper of the VC asset class, nor the control status of venture capitalists. As for the hazard of failure, it is assessed only indirectly, which yields less reliable estimates of the risk of VC.[2]

We propose a novel, non-CAPM-based model that transposes the illiquidity, hazard of failure, and control status of the VC asset class directly into an aggregate risk premium estimate. By construction, the model is clean of the confounding effects of superior fund management. By feeding the model with plausible data, we find that minority investors—the limited partners who passively provide capital to VC funds—should add a premium of 1.29% to public equity's return; whereas control investors—the general partners who manage VC funds—should add only a 0.42% premium.

DRIVERS OF THE VENTURE CAPITAL PREMIUM

Consider three investors: X, Y, and Z. X's equity is 100% allocated to a fund that tracks a broad public stock index, like the Standard & Poor's 500 Index, thus making X a diversified investor. Assuming a risk-free rate of 4% and a required market risk premium of 5%, *X*'s required return on public equity would equal 4% + 5% = 9%. Y's equity is, in turn, 100% invested in a fund that tracks a broad VC index, such as the Cambridge Associates (CA) Index. The CA Index reflects the value of a large number of minority positions in VC funds spanning several industries. Y can, therefore, also be considered a diversified investor—but only within the VC market. Finally, consider Z, a venture capitalist whose equity is 100% invested in a partnership that contains several VC funds, made up of firms scattered across a wide spectrum of industries. Like Y, Z can be deemed a diversified investor within the VC market; but in contrast to Y, Z holds control positions in the private ventures that constitute the VC funds. The central question we address in this article is: What premiums should Y and Z ask over X's required return to get compensated for the risks that arise from investing exclusively in the VC market?

Lack of liquidity and a material hazard of failure have a value-reducing effect on Y's portfolio and, as a consequence, Y must absorb a differential risk premium we will call "required venture capital premium" (RVCP). RVCP is the minimum compensation Y should require on top of public equity returns to be properly rewarded for bearing the unique risks that spring from investing in VC. Z's portfolio is, in turn, as liquid and hazardous as Y's but, in comparison, is made up of control stock positions. As a result, and before being paid compensation for managing the VC partnership, Z must absorb an RVCP composite of illiquidity, hazard, and control, which will be smaller than Y's risk premium—given that the presence of control partly compensates the value-shrinking effects of illiquidity and hazard. Notice that to both Y and Z, who are fully invested in the VC market, RVCP is a systematic risk—one that cannot be diversified away.

To infer the value of RVCP for Y and Z we must first measure the effects of illiquidity, hazard, and control on the value of VC portfolios; and next, translate those effects into risk premiums. In the remainder of this section, we undertake the first task by traveling back in time—to the moment at which companies that are now seasoned and liquid were smaller and private—namely, low-liquidity concerns locked up in the portfolios of VCs.

Illiquidity: From Private to Public

The shares of a quoting company are more liquid than those of a non-quoting one, as they can be rapidly and easily traded in the stock market, with considerable certainty on the realization value, and with minimum transaction costs. The VC market, in contrast, deals in shares of private, non-quoting companies. Selling private shares is difficult, may take a long time, and can even fail to be completed. Private shares are thus much less marketable or liquid—and hence, less valuable or riskier—than those of a similar public company.

Lack of marketability translates into a discount on the price at which shares of a private company are sold as compared with the selling price of public but otherwise similar stock. Panel A of Exhibit 1 synthesizes the evidence on the illiquidity discount coming from studies of firms that evolve from private to public. Virtually all firms in these surveys do not pay dividends and are, therefore, quite relevant to VCs, who usually realize their gains by selling their stock at exit time, not by collecting interim dividends. The 1997–2000 survey comprises only Internet-related firms—a breed of particular interest to VCs who invest in "high-tech" companies that are going public in "hot" markets; this sample includes Amazon.com, whose illiquidity discount climbed to 63%. The sizable average reported for the Internet-related group suggests that, even in a torrid market, investors' enthusiasm for high technology is not enough to curb the noxious impact of lack of marketability on a venture's equity worth. All surveys factored in, the average illiquidity discount turns out to be a highly material figure: 47.3%.

In all fairness, though, only a portion of a VC portfolio should be considered as less-than-fully liquid; some portfolio companies do, indeed, ultimately go

Exhibit 1

Illiquidity Discounts and IPO Percolation Rate

Panel A of this exhibit documents the realized illiquidity discount in the U.S. The discount is calculated as: [(IPO price – Private transaction price)/Private transaction price]. Private transactions occur within five months before the initial public offering (IPO) date in Emory's studies. Emory's 1997–2000 survey comprises dot-com firms only. Data for Emory is a synthesis based on Pratt [2009, Ch. 9 and 10]. Willamette Management Associates data is from Pratt et al. [1996]. Panel B documents the IPO percolation rate (IPO%)—the percentage of exits via IPOs. Percolation rates consider both realized IPOs and IPO registrations. N.A.: not available.

Panel A: Realized Illiquidity Discounts, 1975–2000

Survey	Coverage	Mean Control Premium
Willamette Management Associates	1975–1992	51.4%
Emory	1980–1986	51.0%
	1987–1989	42.0%
	1990–1997	42.0%
	1997–2000	50.0%
	Minimum	42.0%
	Maximum	51.4%
	Mean	47.3%
	Median	50.0%

Panel B: IPO Percolation Rate, 1961–2008

Survey	Coverage	Number of Investments	Number of Exits	IPO Percolation Rate (IPO%)
Gompers and Lerner [1999]	1961–1992	794	491	36.3%
Sahlman [1990]	1980–1988	383	383	4.7%
Das, Jagannathan, and Sarin [2003]	1980–1997	23,208	N.A.	24.4%
Gompers and Lerner [1999]	1983–1994	32,364	25,988	39.0%
Cochrane [2005]	1987–2000	7,765	4,232	46.1%
Hall and Woodward [2010]	1987–2008	22,004	10,992	18.3%
			Minimum	4.7%
			Maximum	46.1%
			Mean	28.1%
			Median	30.4%

through the initial public offering (IPO) hurdle. Panel B of Exhibit 1 suggests that, on average, about 28.1% of the projects in a VC portfolio become eventually public and thus fully liquid. Consequently, portfolio ventures will end up subject only to *a residual illiquidity* discount, or $D_{Residual\ Illiquidity}$, thus:

$$D_{Residual\ Illiquidity} = D_{Realized\ Illiquidity} \cdot (1 - \text{IPO\%}) \qquad (1)$$

where IPO% is the IPO percolation rate.[3] Applying Equation (1) to Exhibit 1 data, the residual illiquidity discount on the average portfolio's value turns out to be $47.3\% \cdot (1 - 28.1\%) = 34\%$.

Hazard: From Stars to Survivors

When VCs are assembling a portfolio of private ventures, each of them is thought to be a winner. Experience will unfortunately prove such belief wrong: By exit time, a material portion of those ventures will have been terminated with partial or total loss to the investor. We call the probability of a venture's failing with losses to VCs over the average holding period *mortality hazard*, *hazard of failure*, or more simply put, *hazard*. The hazard rate is therefore the frequency with which VC-backed firms are expected to be terminated with losses to financial backers over a specific investment horizon.

Panel A of Exhibit 2 documents the available evidence on the five-year realized hazard rate of VC-backed ventures. The 24.4% average is a rule of thumb used by practicing VCs: from one-fourth to one-third of a VC portfolio ultimately will be written off. The hazard figure allows, as well, for an interesting insight: If public firms are on average larger and more seasoned

Exhibit 2

Hazard Rates for Venture Capital-Backed Ventures, 1960–2008

Panel A of this exhibit reports realized hazard rates of U.S. venture capital-backed ventures. *Realized hazard* is defined as the historical frequency with which exited ventures have been terminated with partial or total loss to venture capitalists (VCs) over five years. Panel B reports hazard rates corresponding to three broad stages of venture capital (VC) investment, further broken down into five shorter stages. *Seed capital* funds research aimed at gauging the technical and commercial feasibility of a venture. *Start-up capital* is the capital employed to effectively launch the venture and sustain it through its first year of life. *Third-stage capital* finances expansion—namely, gaining market penetration and ramping up production. *Fourth-stage capital* funds the consolidation of growth. *Exit stage* capital prepares the venture for investors' cash-out, which may take place via a profitable private sale to a strategic investor, an initial public offering, or a buyout by the entrepreneur. The *expected hazard rate* is the risk of loss expected at each stage by 72 venture capital firms, from a survey by Rhunka and Young [1987]. The final *adjusted hazard rate*, deemed to be more realistic than the expected one, is obtained by adjusting the expected hazard as follows: Hazard rate = [Expected hazard rate. (Mean realized hazard rate/Mean expected hazard rate)], whereby the mean realized rate is 24.4%, as per Panel A.

Panel A: Realized Hazard Rates

Survey	Coverage	Number of Investments	Number of Exits	Realized Hazard Rate
Huntsman and Hoban [1980]	1960–1975	110	110	25.0%
Gompers and Lerner [1999]	1961–1992	794	491	25.2%
Sahlman [1990]	1969–1985	383	383	34.5%
Cochrane [2005]	1987–2000	7,765	4,232	16.5%
Gompers and Lerner [1999]	1983–1994	32,364	25,988	23.3%
Hall and Woodward [2010]	1987–2008	22,004	10,992	22.0%
			Minimum	16.5%
			Maximum	34.5%
			Mean	24.4%
			Median	24.1%

Panel B: Hazard Rates per Stage of Investment

Broad Stage	Stage	Expected Hazard Rate	Adjustment Factor	Hazard Rate
Early	Seed	66.2%	1.96	41.5%
	Startup	53.0%	1.57	33.2%
Middle	3rd stage	33.7%	1.00	21.1%
Late	4th stage	20.9%	0.62	13.1%
	Exit stage	20.9%	0.62	13.1%
All stages	Minimum	20.9%		13.1%
	Maximum	66.2%		41.5%
	Mean	38.9%		24.4%
	Median	33.7%		21.1%

than the private firms that compose a VC portfolio, we should expect the hazard rate of the latter to be materially larger than that of the former. A study by Wang [2000], which tracked the fate of 6,128 public U.S. firms over the 1975–1995 period, found a five-year hazard rate of 5.9%. Then hazard in the VC market seems to be about four times that of the public stock market.

But the mean realized hazard reported in Panel A of Exhibit 2 is a central value and therefore, useful only to VCs that plan to inject money in middle-stage/middle-risk ventures. Other VCs may prefer to invest in riskier early-stage ventures—in the seed capital and start-up phases; and still others, in less risky, better-established companies, that are already transitioning towards a private sale or an IPO. To produce data relevant to all types of VCs, we compute in Panel B of Exhibit 2 the hazard rates per investment stage: hazard goes from 41.5%, for very early-stage ventures, down to 13.1%, for exit-stage ventures.

We are now ready to estimate an average hazard-related discount on equity, or D_{Hazard}, which can be expressed as:

$$D_{Hazard} = [1 - (E_{Post\text{-}Hazard}/E_{Pre\text{-}Hazard})] \qquad (2)$$

where $E_{Post\text{-}Hazard}$, or the value of equity adjusted for the probability of failure, can be obtained by simply averaging out equity values for the survival (namely, pre-hazard) and the failure scenario, like this:

$$\text{Equity}_{Post\text{-}Hazard} = (1 - p_{Hazard}) \cdot \text{Equity}_{Survival} + p_{Hazard} \cdot \text{Equity}_{Failure} \tag{3}$$

where p is the hazard rate, or probability of failure with partial or total loss. We will assume the simplifying and conservative assumption that $Equity_{Failure}$ is zero.[4] If failed ventures have zero value, and after some manipulation, it follows that:

$$D_{Hazard} = p_{Hazard} \tag{4}$$

and therefore, the average discount for hazard will be equal to its probability, namely, 24.4%.

Control: From Minority to Dominating Positions

A control shareholding (i.e., holding more than 50% of the shares) is usually less risky than a minority one, since the former carries several control and restructuring privileges that the latter does not. Then a minority interest is worth less than a control interest—or, put differently, the latter is less risky.

When VCs hold control, the value of their shares enjoy a control premium which can be estimated by comparing the share price of the same company in two different situations: when trading in the stock market—where, by definition, only minority positions are negotiated—and when a control position of the same company's stock has been transferred in a private acquisition.[5]

In Exhibit 3, the most reliable estimate of the control premium comes from considering exclusively the last two surveys, which include both positive and negative premiums (control premiums can occasionally be negative); the mean control premium figure is 30%.

Estimating Aggregate Discounts

Using the averages estimated above, we can now compute an aggregate average discount on VC equity for each investor type. For the middle-risk investment stage, for example:

EXHIBIT 3
The Control Premium, 1985–2008

This exhibit documents control premiums for U.S. acquisition deals. A control premium is estimated by comparing the prices of shares of the same company in two different situations: when trading in the stock market—where, by definition, only minority positions are negotiated—and when a control position of the company stock is transferred in a private acquisition. The first four *Mergerstat Review* surveys are biased upward, since they report positive control premiums only. In contrast, the last two *Mergerstat Review* surveys report positive and negative premiums, and are therefore more reliable. All data are from Pratt [2009].

Survey	Coverage	Mean Control Premium
Mergerstat Review (5-day)	1985–1994	39.5%
Mergerstat Review (5-day)	2001–2005	48.9%
Mergerstat BVR 1 (1-day)	2008	39.0%
Mergerstat BVR 2 (2-month)	2008	39.0%
Mergerstat BVR 3 (1-day)	2008	31.0%
Mergerstat BVR 4 (2-month)	2008	29.0%
All surveys	Minimum	29.0%
	Maximum	48.9%
	Mean	37.7%
	Median	39.0%
Last two surveys only	Minimum	29.0%
	Maximum	31.0%
	Mean	30.0%
	Median	30.0%

$$\begin{aligned} D_{Minority\ Investor\ Y} &= \text{Illiquidity Discount} \cdot (1 - \text{IPO\%}) \\ &\quad + \text{Hazard Discount} \\ &= 47.3\% \cdot (1 - 28.1\%) \\ &\quad + 24.4\% = 58.4\% \end{aligned} \tag{5}$$

and:

$$\begin{aligned} D_{Control\ Investor\ Z} &= \text{Illiquidity Discount} \cdot (1 - \text{IPO\%}) \\ &\quad + \text{Hazard Discount} - \text{Control premium} \\ &= 47.3\% \cdot (1 - 28.1\%) \\ &\quad + 24.4\% - 30\% = 28.4\% \end{aligned} \tag{6}$$

We can rely on such aggregate discounts even if illiquidity, control, and hazard interact with each other. For instance, younger firms tend to be more illiquid and more prone to failure, and this positive correlation will increase the standard deviation of the composite (illiquidity-and-hazard) discount, but the composite itself will remain stable. This is always the case when equations that contain interacting inputs take a very simple form—one that includes additions and multiplications only, as in Equations (5) and (6).[6] Our aggregate discounts figures are therefore meaningful and, as we will show momentarily, can work as a solid platform from which to launch the estimation of risk premiums.

TRANSFORMING EQUITY DISCOUNTS INTO RISK PREMIUMS

Now that we have obtained estimates of aggregate discounts per investor type, we need to find a way to transpose them into risk premiums. In Appendix A, we show that the empirical relation between equity discounts and risk premiums can be approximated well by the following equation:

$$RVCP_{NOF} = K \cdot [(1/(1 - D)) - (1 - D)] \qquad (7)$$

where the *NOF* sub-index means that *RVCP* is a net-of-fees premium. In other words, for a passive investor, for example, *RVCP* will be exclusively due to the deleterious effects of illiquidity and hazard; it will not include the expenses and rewards that Y must pay to fund managers like Z, who construct and oversee funds until exit. D is the aggregate discount on equity, and *K*, a coefficient that depends on the growth rate in perpetuity of a VC portfolio's cash flow. In Appendix B, we explain why a 4.2% growth rate is fairly typical of the average U.S. VC portfolio.

Equation (7) can next be put to good use to compute estimates of $RVCP_{NOF}$—which was our original purpose in this article. Exhibit 4 displays the results—namely, the premium benchmarks for the U.S. VC industry. We learn that, for Y-type, passive investors, $RVCP_{NOF}$ is equal to 1.29% across all investment stages. For a control, Z-type investor, $RVCP_{NOF}$ equals 0.42%—as expected, a smaller figure than that for passive investors.

EXHIBIT 4
The Required Venture Capital Premium

This exhibit documents equity discounts and the concomitant net-of-fees required venture capital premiums ($RVCP_{NOF}$) that VCs should require over public equities, based on long-run data about residual illiquidity, hazard, and control. Panel A reports $RVCP_{NOF}$ for a minority investor who, by definition, is subject to illiquidity and hazard only. Panel B reports $RVCP_{NOF}$ for a control investor, who is subject to illiquidity, hazard, and control effects. The mean equity discount is an additive composite of the discount for illiquidity (modulated by the IPO percolation rate), the discount for hazard and, for control investors, the control premium. $RVCP_{NOF}$ figures have been estimated by using a regression equation best fitting the middle broad investment stage (third stage): RVCP = 0.005064. [(1/(1 – D) – (1 – D)]; Adj. $R^2 = 0.996$, whereby *D* is the mean discount on equity. To maintain consistency, the same equation is applied to the other two broad stages. Over the relevant equity discount ranges specified in Panels A and B for each stage, this procedure yields the following mean estimation errors: early stage: –15.7%; middle (third) stage: 7.9%; late stage: 7.5%; all three stages (full sequence): –5.7%. Averages for the *all* category are weighted by the percent of deals per stage—the mean relative concentration of VC investment in each stage, arising from an analysis of 35,587 U.S. VC deals over the 2001–2010 period, as per the Yearbooks of the National Venture Capital Association.

Broad Stage	Stage	Percent of Deals	Mean Equity Discount	$RVCP_{NOF}$: Required Venture Capital Premium Per Stage	$RVCP_{NOF}$: Required Venture Capital Premium Per Broad Stage
Panel A: Minority Investors (Illiquidity and Hazard)					
All	All (weighted average)	100.0%	62.3%	1.29%	1.29%
Early	Seed	9.2%	82.0%	2.72%	1.95%
	Startup	28.1%	72.4%	1.69%	
Middle	3rd. Stage	38.5%	58.4%	1.01%	1.01%
Late	4th. Stage	23.0%	49.1%	0.74%	0.74%
	Exit	1.2%	49.1%	0.74%	
Panel B: Control Investors (Illiquidity, Hazard, and Control)					
All	All (weighted average)	100.0%	32.3%	0.42%	0.42%
Early	Seed	9.2%	52.0%	0.81%	0.64%
	Startup	28.1%	42.4%	0.59%	
Middle	3rd. Stage	38.5%	28.4%	0.34%	0.34%
Late	4th. Stage	23.0%	19.1%	0.22%	0.22%
	Exit	1.2%	19.1%	0.22%	

ASSESSING VENTURE CAPITAL PERFORMANCE

VC is a friction-rich environment where investors sink money into low-liquidity ventures in which they hold different degrees of control, being exposed to the material chance that, in a coup of Darwinian selection, such investments vaporize partially or totally after a small number of years. We capture the effect of such frictions with a new empirical approach—the required venture capital premium (RVCP) model.

As embodied in Equation (7), the RVCP model is a straightforward tool to determine minimum performance benchmarks and, by extension, the risk-adjusted performance of venture capital. Using their own estimates of residual illiquidity, control, and hazard, VC investors can use Equation (7) to calculate RVCPs, add them to public equity's return, and compare the resulting figure against the historical performance of single projects or portfolios; the procedure will show, at a glance, whether the investment in question has created, preserved, or destroyed economic value.

The RVCP model is an empirically grounded alternative to the rules of thumb frequently used in the estimation of target premiums for venture capital investments. Passive institutional investors, for example, profoundly distrust the risk estimates yielded by CAPM-based regressions, and keep adding arbitrary premiums—say, 3% to 5%—to public market returns. Being based on actual data on illiquidity, control, and hazard, and clean of the confounding effects of superior fund management, our RVCP estimates are probably more intuitive and plausible.

As for those analysts at VC funds who keep discounting cash flows at arbitrarily defined hurdle internal rates of returns (IRRs) (implicitly assuming, without evidence, that such IRRs are higher than minimum yet unknown benchmark returns), they may instead start discounting at a return that contains the RVCP, truly reflecting the effects of a venture's illiquidity and hazard. Relying on RVCP benchmarks will likely help general partners (GPs) to simultaneously deflect the winner and loser curses, i.e., avoid overpaying for targets and avoid underpricing them. In passing, RVCP benchmarks will reveal whether the IRRs used by GPs are really excessive—as entrepreneurs frequently complain.

APPENDIX A

THE REQUIRED VENTURE CAPITAL PREMIUM MODEL

Equity discounts are adjustments that apply directly to a venture's equity value—namely, they are not risk premiums. To obtain premiums, we must be able to express the value of a private equity investment as a function of its opportunity cost of capital. Here is how we do it: Any discount D on a venture's equity can be expressed as:

$$D = [1 - (E_{Post\text{-}Discount}/E_{Pre\text{-}Discount})] \qquad \text{(A-1)}$$

where E stands for the venture's equity value. The familiar discounted cash-flow formula can be used to compute E before and after the application of a discount, thus:

$$D = 1 - \frac{\sum_{i=1}^{n} \frac{FCFE_i}{(1+C_{E1})^i} + \frac{\frac{FCFE_n \cdot (1+g_\infty)}{(C_{E1}-g_\infty)}}{(1+C_{E1})^n}}{\sum_{i=1}^{n} \frac{FCFE_i}{(1+C_{E0})^i} + \frac{\frac{FCFE_n \cdot (1+g_\infty)}{(C_{E0}-g_\infty)}}{(1+C_{E0})^n}} \qquad \text{(A-2)}$$

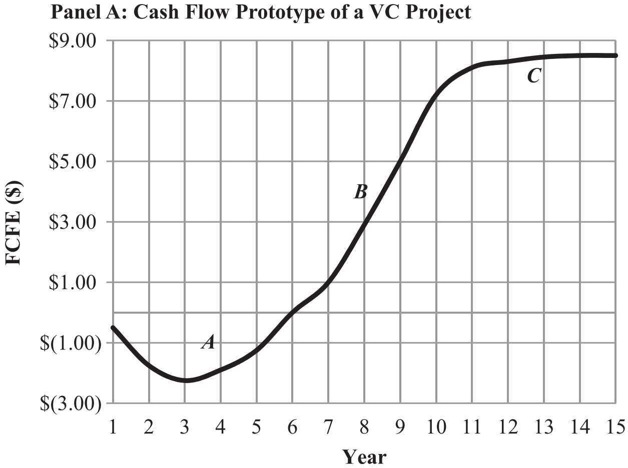
where C_{E0} and C_{E1} are, respectively, the opportunity costs before and after applying discount D on equity, and being $C_{E1} > C_{E0}$. In the right side of Equation (A-2), the second term is the quotient of the values of the venture's equity. The first term in the quotient's numerator is the venture's present value in the holding period n, being $FCFE_i$ the venture's free-cash-flow-to-equity in period i; the second term is the venture's terminal value, being g_∞ the cash flow's growth rate in perpetuity. Since a risk premium, or RP, is equal to $(C_{E1} - C_{E0})$, Equation (A-2) allows us to draw the empirical relation between equity discounts and risk premiums. To do so, we define three parameters: first, the typical shape or structure over time of a venture's cash flow, which experience shows is typically S-shaped—VCs call it "J-curve" or "hockey-stick'" pattern—a path along which cash flows start negative and take a dip before going up (see Panel A in Exhibit A1). Second, the length of the VC's holding horizon; the typical time span in VC is five years. Last is the value of g_∞, the perpetual growth rate of the last cash flow in the holding period, which in Appendix B we estimated at 4.2% for the U.S. VC industry. The empirical relation between value and premium turns out to take the shape shown in Panel B, to which we fit a statistically significant equation. And here is the prize: By defining an

Exhibit A1

Cash Flow Prototype and the Empirical Link between Equity Discounts and Risk Premiums

Panel A of this exhibit depicts a free-cash-flow-to-equity (FCFE) prototype for a hypothetical yet typical venture capital-backed project, since inception to maturity. Section *A* is the typical cash flow structure of an early-stage venture, while Section *C* is the norm for late-stage ventures—namely, those already throwing off positive cash flows and approaching a situation of stable growth. Section *B* is an intermediate instance and reflects the cash flow pattern of a middle-stage/middle-risk venture. The investment horizon for each stage is set at five years. Panel B shows the empirical relation between discount and risk premium as obtained by simulation that uses Equation (A-2), for the *B* section of the cash flow. The "OBS" curve plots the actual relation between observed net-of-fees risk premium, *RP*, and the discount on a venture's equity, *D*. *RP* equals ($C_{E1} - C_{E0}$), whereby C_{E0} (C_{E1}) is the venture's equity value before (after) applying discount *D* to equity. The "EST" curve depicts the estimated (predicted) relation between discount and net-of-fees risk premium using a strictly monotonic increasing function that is approximated well by the following homographic polynomial, in which the risk premium asymptotically approaches $+\infty$ when the discount approaches 100%: $\text{RVCP}_{NOF} = 0.005064 \cdot [((1/(1-D)) - (1-D)]$. The regression is highly significant (Adjusted R^2: 0.996; *t*-coefficient: 140.16; *p*-value: 0.000). The fitted curve under- or overstates the actual premium depending on the size of the discount, yet at 8.5%, the mean absolute error is quite acceptable for estimation purposes. Robustness tests show that the curve's *shape* is invariant with respect to a cash flow's structure and perpetual growth rate. The equation's *coefficient* is invariant with respect to the cash flow, but dependent on its growth rate; see Appendix B for details.

Panel A: Cash Flow Prototype of a VC Project



Panel B: The Relation Between Discounts and Risk Premiums

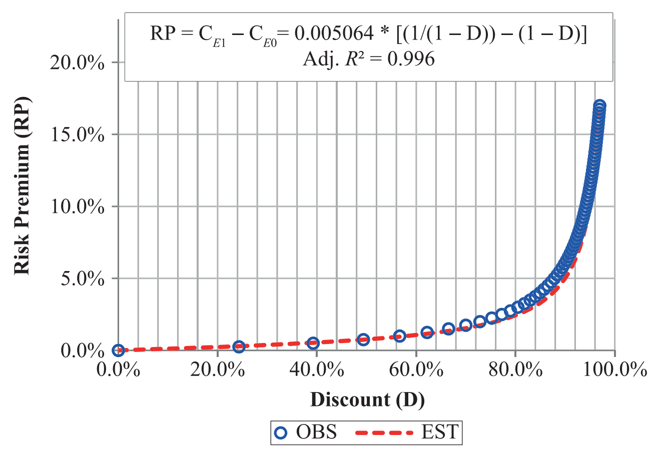


Exhibit A2

Required Venture Capital Premiums at Various Cash Flow Growth Rates

Growth Rate in Perpetuity of Portfolio's Cash Flow	Required Venture Capital Premium	
	Minority Investor	**Control Investor**
1.0%	5.73%	1.89%
2.0%	4.51%	1.48%
3.0%	3.16%	1.04%
4.0%	1.67%	0.55%
4.2%	1.29%	0.42%
5.0%	0.05%	0.02%

actual or expected growth rate for a specific portfolio or individual venture, general and limited partners may infer from Exhibit A2 the required premium that should be used to assess investment performance.

Appendix B

THE PERPETUAL GROWTH RATE OF CASH FLOW IN A VC PORTFOLIO

The growth rate in perpetuity of the cash flow of the average U.S. VC portfolio, g_∞, can be computed as a weighted average of the dollar volume allocated to the sectors that compose the portfolio, and $g_{\infty\ Sector}$, the perpetual growth rate of the sectors' cash flows. To compute the former we rely on data from the National Venture Capital Association (see Exhibit B1). Finding out $g_{\infty\ Sector}$ is more challenging: using historical growth figures is impractical, for aggregate cash flows are sometimes negative, thus rendering averages non-meaningful. We rely instead on the following economic intuition: First, $g_{\infty\ Sector}$ will depend on the growth potential of a sector's aggregate demand: it is only natural to expect that, in the foreseeable future, the demand for products and

EXHIBIT B1

The Perpetual Growth Rate of the Cash Flow in the Typical U.S. Venture Capital Portfolio

Panel A of this exhibit depicts the composition of the typical venture capital (VC) portfolio in the U.S. Industry data come from the Yearbooks of the National Venture Capital Association for the period 2001–2010. Deals in the categories "Financial Services" and "Other" (2.5% of the total) have been excluded to obtain a close industry match across panels A and B. Estimates of $g_{\infty\ Sector}$ are taken directly from Panel B. Average g_∞ is the growth rate of the typical VC portfolio, weighted by the dollar volume invested per industry. Panel B displays $g_{\infty\ Sector}$, the perpetual growth rate of cash flows per sector, already scaled as to obtain an implied gross domestic product (GDP) growth rate over 500 years of 7.0% per annum—a number approximately equal to the growth of the U.S. economy over the 1960–2009 period. Industry cash flows are proxied by after-tax operating income. *ROCE* is Return on Capital Employed, defined as: [Net Operating Profits after Taxes]/[Net Fixed assets + Operating Working Capital]. Annual revenue growth and *ROCE* are computed on 6,666 U.S. firms based on Bloomberg and Value Line data. In Panel B, industries are consolidated as necessary to match the categories in Panel A, by averaging out the elementary $g_{\infty\ Sector}$ figures. Growth figures by sector are obtained by multiplying the product of revenue growth and ROCE by 0.879, a scaling factor; this assures that aggregate corporate cash flows grow at the rhythm of U.S. GDP.

Panel A: Sector Breakdown and Perpetual Growth Rate of the Cash Flow in the Average U.S. Venture Capital Portfolio

Sector	# Deals	$	% Deals	% $	$g_{\infty sector}$
Biotechnology	3,798	36,351,047,900	11.9%	16.2%	4.8%
Business Products and Services	942	4,696,949,300	2.9%	2.1%	2.9%
Computers and Peripherals	604	4,461,635,100	1.9%	2.0%	2.5%
Consumer Products and Services	801	3,399,361,100	2.5%	1.5%	1.2%
Electronics/Instrumentation	711	4,058,249,600	2.2%	1.8%	2.3%
Healthcare Services	582	2,781,433,500	1.8%	1.2%	2.1%
Industrial/Energy	2,039	18,141,562,400	6.4%	8.1%	2.9%
IT Services	2,148	13,543,093,700	6.7%	6.0%	6.7%
Media and Entertainment	2,490	13,484,308,300	7.8%	6.0%	1.6%
Medical Devices and Equipment	2,879	22,269,939,100	9.0%	9.9%	1.9%
Networking and Equipment	1,574	16,675,136,800	4.9%	7.4%	2.0%
Retailing/Distribution	395	2,004,583,300	1.2%	0.9%	2.2%
Semiconductors	1,796	15,173,490,000	5.6%	6.7%	5.6%
Software	8,849	49,479,692,600	27.7%	22.0%	6.7%
Telecommunications	2,331	18,354,137,000	7.3%	8.2%	4.0%
Grand Total	31,939	224,874,619,700	100.0%	100.0%	
			Simple average g_∞		3.3%
			Weighted average g_∞		4.2%

Panel B: Perpetual Growth Rate of Cash Flows per Sector

Sector	Revenue Growth	ROCE	Sales Growth X ROCE	$g_{\infty Sector}$
Advertising	9.94%	11.92%	1.18%	1.04%
Aerospace/Defense	20.45%	17.43%	3.56%	3.13%
Air Transport	16.82%	33.07%	5.56%	4.89%
Apparel	14.17%	15.54%	2.20%	1.94%
Auto & Truck	3.31%	15.65%	0.52%	0.46%
Auto Parts	5.23%	25.26%	1.32%	1.16%
Beverage	3.93%	18.72%	0.74%	0.65%
Biotechnology	32.92%	14.57%	4.80%	4.22%
Building Materials	12.50%	16.15%	2.02%	1.77%
Cable TV	9.97%	9.79%	0.98%	0.86%
Chemical (Basic)	4.36%	19.93%	0.87%	0.76%
Chemical (Diversified)	17.26%	25.23%	4.35%	3.83%
Chemical (Specialty)	13.65%	18.77%	2.56%	2.25%
Coal	14.04%	27.03%	3.79%	3.33%
Computer Software/Svcs	20.80%	36.57%	7.61%	6.69%
Computers/Peripherals	6.96%	31.61%	2.20%	1.93%
Drug	25.04%	24.85%	6.22%	5.47%

Exhibit B1 (*Continued*)

Panel B: Perpetual Growth Rate of Cash Flows per Sector

Sector	Revenue Growth	ROCE	Sales Growth X ROCE	$g_{\infty Sector}$
E-Commerce	13.72%	15.89%	2.18%	1.92%
Educational Services	4.07%	56.00%	2.28%	2.00%
Electric Utility	2.80%	12.42%	0.35%	0.31%
Electrical Equipment	4.81%	17.59%	0.85%	0.74%
Electronics	13.85%	13.06%	1.81%	1.59%
Entertainment	14.09%	9.77%	1.38%	1.21%
Environmental	10.92%	12.60%	1.38%	1.21%
Food Processing	9.63%	16.12%	1.55%	1.36%
Food Wholesalers	25.71%	11.53%	2.96%	2.61%
Furn/Home Furnishings	3.18%	14.87%	0.47%	0.42%
Grocery	22.22%	16.63%	3.70%	3.25%
Heavy Construction	13.75%	19.47%	2.68%	2.35%
Homebuilding	2.30%	7.42%	0.17%	0.15%
Hotel/Gaming	19.31%	11.03%	2.13%	1.87%
Household Products	9.64%	16.59%	1.60%	1.41%
Human Resources	34.56%	17.42%	6.02%	5.29%
Industrial Services	15.13%	17.62%	2.67%	2.34%
Information Services	13.78%	18.66%	2.57%	2.26%
Internet	15.95%	21.77%	3.47%	3.05%
Machinery	13.52%	16.46%	2.22%	1.96%
Maritime	9.99%	13.33%	1.33%	1.17%
Medical Services	12.68%	18.48%	2.34%	2.06%
Medical Supplies	10.70%	20.08%	2.15%	1.89%
Metal Fabricating	26.45%	21.52%	5.69%	5.00%
Metals & Mining (Div.)	83.21%	29.22%	8.69%	7.64%
Natural Gas Utility	1.78%	11.77%	0.21%	0.18%
Newspaper	–3.41%	10.45%	–0.40%	–0.35%
Office Equip/Supplies	3.30%	17.67%	0.58%	0.51%
Oilfield Svcs/Equip.	38.08%	22.82%	8.69%	7.64%
Packaging & Container	4.88%	16.34%	0.80%	0.70%
Paper/Forest Products	0.02%	14.59%	0.00%	0.00%
Petroleum (Integrated)	12.48%	26.42%	3.30%	2.90%
Petroleum (Producing)	20.44%	22.85%	4.67%	4.11%
Pharmacy Services	31.86%	6.29%	2.00%	1.76%
Power	32.46%	12.43%	4.03%	3.55%
Precision Instrument	18.48%	13.55%	2.50%	2.20%
Property Management	20.29%	11.28%	2.29%	2.01%
Publishing	18.00%	21.79%	3.92%	3.45%
Railroad	14.34%	14.26%	2.05%	1.80%
Recreation	23.81%	15.04%	3.58%	3.15%
Restaurant	13.10%	21.02%	2.75%	2.42%
Retail Automotive	11.20%	12.90%	1.44%	1.27%
Retail Building Supply	10.32%	19.71%	2.03%	1.79%
Retail Store	16.28%	16.54%	2.69%	2.37%
Semiconductor	15.06%	27.92%	4.20%	3.70%
Semiconductor Equip	28.61%	29.78%	8.52%	7.49%
Shoe	18.65%	29.57%	5.52%	4.85%
Steel	11.77%	29.51%	3.47%	3.05%
Telecom. Equipment	16.71%	28.95%	4.84%	4.25%
Telecom. Services	26.57%	15.72%	4.18%	3.67%
Tobacco	–4.96%	27.06%	–1.30%	–1.14%
Toiletries/Cosmetics	2.79%	21.47%	0.60%	0.53%
Trucking	9.24%	18.57%	1.72%	1.51%
Water Utility	2.97%	9.47%	0.28%	0.25%
Wireless Networking	23.76%	9.38%	2.23%	1.96%
	Scaling factor			0.879
	Implied GDP Growth			7.00%

services of younger industries (like software or biotechnology) will be higher than that for declining ones (e.g., traditional newspapers or tobacco). Second, $g_{\infty\ Sector}$ will be a function of the return on capital employed (*ROCE*) achieved in the sector. Mauboussin and Johnson [1997] have argued that firms with larger *ROCE* figures are those better able to shield their profits from the competition. A sector with a higher *ROCE* will likewise be able to sustain a positive cash flow growth for longer periods. Then the larger the expected revenue growth *and ROCE* of an industry, the larger its $g_{\infty\ Sector}$ will be. In Exhibit B1 we calculate $g_{\infty\ Sector}$ as the product of revenue growth and *ROCE*, and next we *scale* the resulting figures so that the weighted average total complies with an empirical regularity reported by Chan et al. [2003]: over the long term, the growth rate of the aggregate corporate cash flow of public companies closely matches that of the nominal gross domestic product (GDP). Shown in Panel B, our $g_{\infty\ Sector}$ estimates have therefore two crucial virtues: they reflect the differentials of sustainable demand and profits across sectors and, at the same time, do not violate the empirical constraint that aggregate cash flows must grow at the economy's rhythm. Back to Panel A, we find that, at an assumed economy's growth rate of 7%, the cash flow of the average U.S. VC portfolio turns out to grow at 4.2%; this is the value we have used in the construction of the suite of premiums appearing in Exhibit 4.

ENDNOTES

[1]We use the term *venture capital* (VC) to designate investments in early- (venture capital proper), middle, or late-stage (private equity proper) private companies under direct management of VC-backed entrepreneurs. Buyout funds are thus excluded from the definition. *Venture capitalists* (VCs) are the general partners, namely, the individuals who assemble and manage VC funds funded by limited partners—passive providers of capital who pay general partners a compensation for their management efforts.

[2]The CAPM was developed by Sharpe [1964]. CAPM-based models applied to VC include: Ljungkvist and Richardson [2003]; Korteweg and Sorensen [2010]; and Driessen et al. [2012]. All CAPM-based model measure hazard indirectly: their betas are correlated to firm size, which is in turn inversely correlated to a firm's probability of failure.

[3]Some minority investors (like pension funds) are natural buy-and-hold agents and may not *care* about the illiquidity of their venture capital holdings, but this does not detract from the fact that the bulk of such holdings will forever remain less liquid than public stock—for most ventures will never cross the IPO threshold.

[4]The assumption does not detract from the generality of Equation (3), inasmuch as $Equity_{Failure}$ can always be expressed as a fraction of $Equity_{Survival}$.

[5]VCs may benefit from the benefits of control without having a majority position in a portfolio company, inasmuch as the term sheet binding VCs and the entrepreneur embodies control clauses (e.g., the right to shut down funding to the venture or to fire the entrepreneur), which do not depend on the percentage held by VCs. Nevertheless, we will assume that VCs' private benefits of control are properly accounted for by the empirical value differentials between majority and minority shareholdings.

[6]The same phenomenon is well known in portfolio theory: in a multiple-asset portfolio, the weighted average of the portfolio's return, which is a linear combination of the returns of each asset, will be about the same whether individual asset returns are correlated or not.

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